INVESTIGATION 2-A

The Components of Projectile Motion

TARGET SKILLS

- Analyzing and interpreting
- Modelling concepts
- Communicating results

A heavy steel ball rolling up and down a ramp follows the same type of trajectory that a projectile follows. You will obtain a permanent record of the steel ball's path by placing a set of white paper and carbon paper in its path. You will then analyze the vertical and horizontal motion of the ball and find mathematical relationships that describe the path.

Problem

What patterns exist in the horizontal and vertical components of projectile velocity?

Hypothesis

Formulate a hypothesis about the relationships between time and the vertical distance travelled by the steel ball.

Equipment

- large sheet of plywood
- very heavy steel ball
- metre stick, graph paper, tape
- set of white paper and carbon paper (or pressure-sensitive paper)

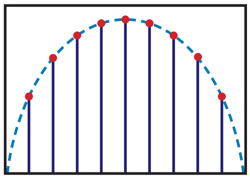
Procedure

1. Set up the apparatus as illustrated.
   CAUTION Wear impact-resistant safety goggles. Also, do not stand close to other people or equipment while doing these activities.

2. Practise rolling the steel ball up the slope at an angle, so that it follows a curved path that will fit the size of your set of white paper and carbon paper.
3. Tape the carbon paper and white paper onto the plywood so that, when the steel ball rolls over it, the carbon paper will leave marks on the white paper.
4. Roll the steel ball up the slope at an angle, as you practised, so that it will roll over the paper and leave a record of its path.
5. Remove the white paper from the plywood. Draw approximately nine or more equally spaced lines vertically through the trajectory.

Analyze and Conclude

1. Measure the vertical displacement in each segment of the path of the steel ball, as shown in the diagram.



2. Assuming that the motion of the ball was uniform in the horizontal direction, each equally spaced vertical line represents the same amount of time. Call it one unit of time.
3. Separate your data into two parts: (a) the period of time that the ball was rolling upward and (b) the period of time that the ball was rolling downward. For each set of data, make a graph of vertical-distance-versus-time units.
4. Use curve-straightening techniques to convert your graphs to straight lines. (See Skill Set 4.)
5. Write equations to describe your graphs.
6. Is the vertical motion of the steel ball uniform or uniformly accelerated?
7. How does it compare to the vertical motion of a freely falling object?
8. Was your hypothesis valid or invalid?
9. Is this lab an appropriate model for actual projectile motion? Explain why or why not.

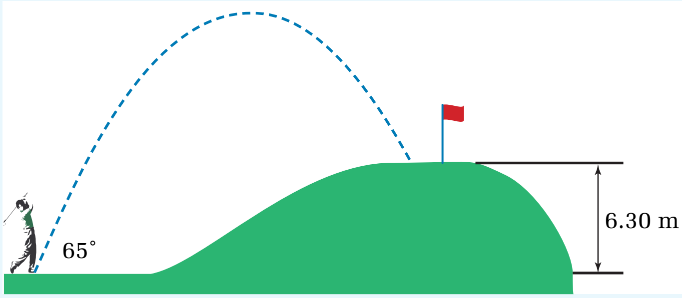
Analyzing Parabolic Trajectories

1. **A golfer hits the golf ball off the tee, giving it an initial velocity of 32.6 m/s at an angle of 65° with the horizontal. The green where the golf ball lands is 6.30 m higher than the tee, as shown in the illustration. Find**

(a) the time interval during which the golf ball was in the air

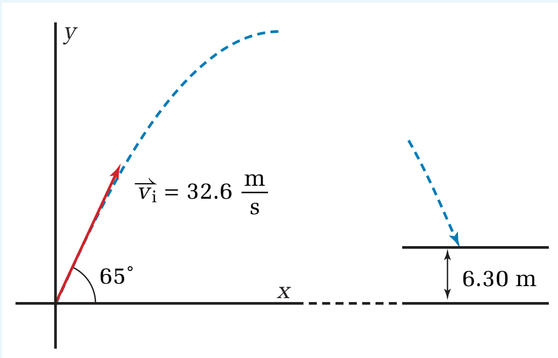
(b) the horizontal distance that it travelled

(c) the velocity of the ball just before it hit the ground (neglect air friction)



Conceptualize the Problem

- Start to frame the problem by making a sketch that includes a *coordinate system*, the *initial conditions*, and all of the known information.
- The golf ball has a *positive initial velocity* in the *vertical* direction. It will rise and then fall according to the kinematic equations.
- The *vertical acceleration* of the golf ball is *negative* and has the *magnitude* of the acceleration due to *gravity*.
- The *time interval* is determined by the *vertical* motion. The *time interval ends* when the golf ball is at a height equal to the *height of the green*.
- The golf ball will be at the height of the green *twice*, once while it is *rising* and once while it is *falling*.
- Motion in the *horizontal* direction is *uniform*; that is, it has a *constant velocity*.
- The horizontal *displacement of the ball* depends on the *horizontal component* of the *initial velocity* and on the duration of the flight.



Identify the Goal

(a) The time interval, Δt, that the golf ball was in the air

(b) The horizontal distance, Δx, that the golf ball travelled

(c) The final velocity of the golf ball, $\vec{v}_f$

Identify the Variables

Known		Implied	Unknown	
$\lvert\vec{v}_i\rvert = 32.6\ \frac{m}{s}$	$\Delta y = 6.30\ m$	$a_y = -9.81\ \frac{m}{s^2}$	Δt	$\vec{v}_f$
$\theta_i = 65°$			Δx	θ_f
			v_{ix}	v_{iy}

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Develop a Strategy

Find the horizontal and vertical components of the initial velocity.

$v_{ix} = |\vec{v}_i| \cos\theta$ $\quad$ $v_{iy} = |\vec{v}_i| \sin\theta$

$v_{ix} = 32.6 \frac{m}{s} \cos 65°$ $\quad$ $v_{iy} = 32.6 \frac{m}{s} \sin 65°$

$v_{ix} = 13.78 \frac{m}{s}$ $\quad$ $v_{iy} = 29.55 \frac{m}{s}$

Find the time interval at which the ball is at a vertical position of 6.30 m by using the kinematic equation that relates displacement, initial velocity, acceleration, and the time interval. You cannot solve directly for the time interval, because you have a quadratic equation. Substitute in the numerical values.

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y\Delta t^2$$

$$6.30 \text{ m} = 29.55 \frac{m}{s}\Delta t + \frac{1}{2}\left(-9.81 \frac{m}{s^2}\right)\Delta t^2$$

Rearrange the equation into the general form of a quadratic equation and solve using the quadratic formula $\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$$4.905\Delta t^2 - 29.55\Delta t + 6.30 = 0$$

$$\Delta t = \frac{29.55 \pm \sqrt{(29.55)^2 - 4(4.905)(6.30)}}{2(4.905)}$$

$$\Delta t = \frac{29.55 \pm \sqrt{749.597}}{9.81}$$

$$\Delta t = 0.2213 \text{ s (or) } 5.803 \text{ s}$$

$$\Delta t \cong 5.8 \text{ s}$$

(a) The smaller value is the time that the ball reached a height of 6.30 m when it was rising. The golf ball hit the green 5.8 s after it was hit off the tee.

Use 5.803 s and the equation for constant velocity to determine the horizontal distance travelled by the golf ball.

$$v = \frac{\Delta x}{\Delta t}$$

$$\Delta x = v\Delta t$$

$$\Delta x = \left(13.78 \frac{m}{\cancel{s}}\right)(5.803 \cancel{s})$$

$$\Delta x = 79.965 \text{ m}$$

$$\Delta x \cong 8.0 \times 10^1 \text{ m}$$

(b) The golf ball travelled 80 m in the horizontal direction.

Find the vertical component of the final velocity by using the kinematic equation that relates the initial and final velocities to the acceleration and the time interval.

$$v_{fy} = v_{iy} + a_y\Delta t$$

$$v_{fy} = 29.55 \frac{m}{s} + \left(-9.81 \frac{m}{s^{\cancel{2}}}\right)(5.803 \cancel{s})$$

$$v_{fy} = -27.38 \frac{m}{s}$$

Use the Pythagorean theorem to find the magnitude of the final velocity.

$$|\vec{v}_f| = \sqrt{\left(13.78\ \frac{m}{s}\right)^2 + \left(-27.38\ \frac{m}{s}\right)^2}$$

$$|\vec{v}_f| = \sqrt{939.55\ \frac{m^2}{s^2}}$$

$$|\vec{v}_f| = 30.65\ \frac{m}{s}$$

$$|\vec{v}_f| \cong 31\ \frac{m}{s}$$

Use trigonometry to find the angle that the final velocity makes with the horizontal.

$$\tan\theta = \left(\frac{v_{fy}}{v_x}\right)$$

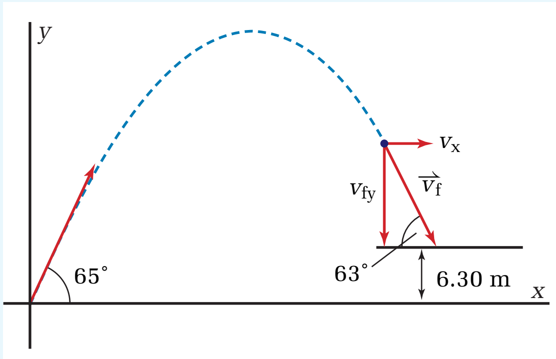
$$\theta = \tan^{-1}\left(\frac{v_{fy}}{v_x}\right)$$

$$\theta = \tan^{-1}\frac{|-27.38\ \frac{m}{s}|}{|13.78\ \frac{m}{s}|}$$

$$\theta = \tan^{-1} 1.9869$$

$$\theta = 63.28^\circ$$

$$\theta \cong 63^\circ$$



(c) The final velocity of the golf ball just before it hit the ground was 31 m/s at 63° with the horizontal.

Validate the Solution

Since the golf ball hit the ground at a level slightly higher than the level at which it started, you would expect the final velocity to be slightly smaller than the initial velocity and the angle to be a little smaller than the initial angle. These results were obtained. All of the units cancelled properly.

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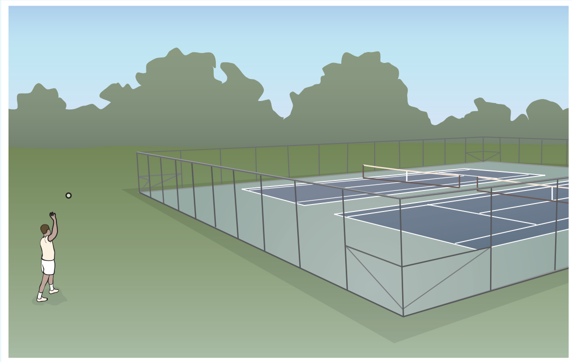
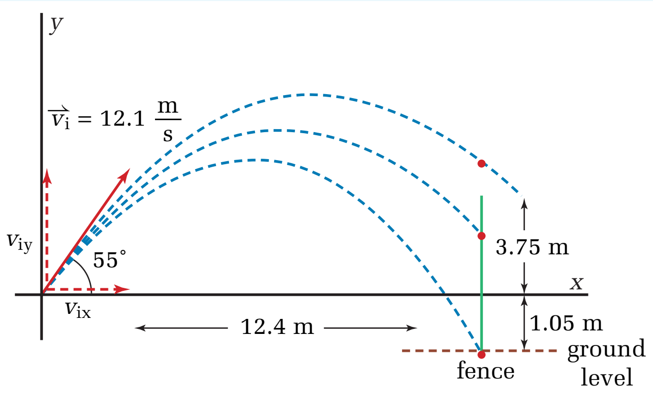
2. You are playing tennis with a friend on tennis courts that are surrounded by a 4.8 m fence. Your opponent hits the ball over the fence and you offer to retrieve it. You find the ball at a distance of 12.4 m on the other side of the fence. You throw the ball at an angle of 55.0° with the horizontal, giving it an initial velocity of 12.1 m/s. The ball is 1.05 m above the ground when you release it. Did the ball go over the fence, hit the fence, or hit the ground before it reached the fence? (Ignore air friction.)



Conceptualize the Problem

- Make a sketch of the *initial conditions* and the three options listed in the question.
- Choose the *origin* of the coordinate system to be at the point at which the *ball left your hand*.
- The equations for *uniformly accelerated motion* apply to the *vertical* motion.
- The definition for *constant velocity* applies to the *horizontal* motion.
- Because the *x*-axis is above ground level, you will have to determine where the top of the fence is relative to the *x*-axis.
- The *time interval* is the *link* between the *vertical* motion and the *horizontal* motion. Finding the time interval required for the ball to reach the position of the fence will allow you to determine the *height* of the ball when it reaches the fence.



Identify the Goal

Whether the ball went over the fence, hit the fence, or hit the ground before reaching the fence

Identify the Variables

Known		Implied	Unknown	
$\lvert\vec{v}_i\rvert = 12.1\ \frac{m}{s}$	$\Delta x = 12.4$ m	$a_y = -9.81\ \frac{m}{s^2}$	Δt	$\vec{v}_{iy}$
$\theta = 55°$	$h = 4.8$ m		$\vec{v}_{ix}$	Δy

Develop a Strategy

Find the *x*- and *y*-components of the initial velocity.

$$v_{ix} = \lvert\vec{v}_i\rvert \cos\theta \qquad v_{iy} = \lvert\vec{v}_i\rvert \sin\theta$$

$$v_{ix} = 12.1\ \frac{m}{s} \cos 55° \qquad v_{iy} = 12.1\ \frac{m}{s} \sin 55°$$

$$v_{ix} = 6.940\ \frac{m}{s} \qquad v_{iy} = 9.912\ \frac{m}{s}$$

To find the time interval, use the equation for the definition of constant velocity and the data for motion in the horizontal direction.

$$v_x = \frac{\Delta x}{\Delta t}$$

$$\Delta t = \frac{\Delta x}{v_x}$$

$$\Delta t = \frac{12.4\ \cancel{\text{m}}}{6.940\ \frac{\cancel{\text{m}}}{\text{s}}}$$

$$\Delta t = 1.787\ \text{s}$$

To find the height of the ball at the time that it reaches the fence, use the kinematic equation that relates displacement, acceleration, initial velocity, and time interval.

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y\Delta t^2$$

$$\Delta y = \left(9.912\ \frac{\text{m}}{\cancel{\text{s}}}\right)(1.787\ \cancel{\text{s}}) + \frac{1}{2}\left(-9.81\ \frac{\text{m}}{\cancel{\text{s}^2}}\right)(1.787\ \cancel{\text{s}})^2$$

$$\Delta y = 2.05\ \text{m}$$

Determine the position of the top of the fence in the chosen coordinate system.

$$y_{\text{fence}} = h - y_{\text{ground to } x\text{–axis}}$$

$$y_{\text{fence}} = 4.8\ \text{m} - 1.05\ \text{m}$$

$$y_{\text{fence}} = 3.75\ \text{m}$$

The ball hit the fence. The fence is 3.75 m above the horizontal axis of the chosen coordinate system, but the ball was only 2.05 m above the horizontal axis when it reached the fence.

Validate the Solution

The units all cancel correctly. The time of flight (about 1.8 s) and the height of the ball (about 2 m) are reasonable values.

PRACTICE PROBLEMS

9. While hiking in the wilderness, you come to the top of a cliff that is 60.0 m high. You throw a stone from the cliff, giving it an initial velocity of 21 m/s at 35° above the horizontal. How far from the base of the cliff does the stone land?

10. A batter hits a baseball, giving it an initial velocity of 41 m/s at 47° above the horizontal. It is a home run, and the ball is caught by a fan in the stands. The vertical component of the velocity of the ball when the fan caught it was –11 m/s. How high is the fan seated above the field?

11. During baseball practice, you go up into the bleachers to retrieve a ball. You throw the ball back into the playing field at an angle of 42° above the horizontal, giving it an initial velocity of 15 m/s. If the ball is 5.3 m above the level of the playing field when you throw it, what will be the velocity of the ball when it hits the ground of the playing field?

12. Large insects such as locusts can jump as far as 75 cm horizontally on a level surface. An entomologist analyzed a photograph and found that the insect's launch angle was 55°. What was the insect's initial velocity?

You have learned to make predictions about projectile motion by doing calculations, but can you make any predictions about patterns of motion without doing calculations? In the following Quick Lab, you will make and test some qualitative predictions.

QUICK LAB

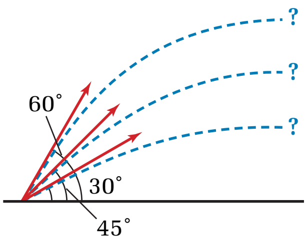
Maximum Range of a Projectile

TARGET SKILLS

- Initiating and planning
- Predicting
- Performing and recording

Football punters try to maximize “hang time” to give their teammates an opportunity to rush downfield while the ball is in the air. Small variations in the initial velocity, especially the angle, make the difference between a great kick and good field position for the opposition.

What launch angle above the horizontal do you predict would maximize the range of an ideal projectile? Make a prediction and then, if your school has a projectile launcher, test your prediction by launching the same projectile several times at the same speed, but at a variety of different angles. If you do not have a projectile launcher, try to devise a system that will allow you to launch a projectile consistently with the same speed but at different angles. Carry out enough trials so you can be confident that you have found the launch angle that gives the projectile the longest range. Always consult with your teacher before using a launch system.



Analyze and Conclude

1. What effect do very large launch angles have on the following quantities?
   (a) maximum height
   (b) vertical velocity component
   (c) horizontal velocity component
   (d) range
2. What effect do very small launch angles have on the above quantities?
3. Did you see any patterns in the relationship between the launch angle and the range of the projectile? If so, describe these patterns.
4. How well did your experimental results match your prediction?
5. What factors might be causing your projectile to deviate from the ideal?
6. Suppose your experimental results were quite different from your prediction. In which number would you place more confidence, your theoretical prediction or your experimental results? Why?

Symmetrical Trajectories

If a projectile lands at exactly the same level from which it was launched and air friction is neglected, the trajectory is a perfectly symmetrical parabola, as shown in Figure 2.4. You can derive some general relationships that apply to all symmetrical trajectories and use them to analyze these trajectories. Follow the steps in the next series of derivations to see how to determine the time of flight, the range, and the maximum height for projectiles that have symmetrical trajectories.

ELECTRONIC LEARNING PARTNER

Enhance your knowledge and test your predictive skills by doing the projectile motion interactive activity provided by your Electronic Learning Partner.

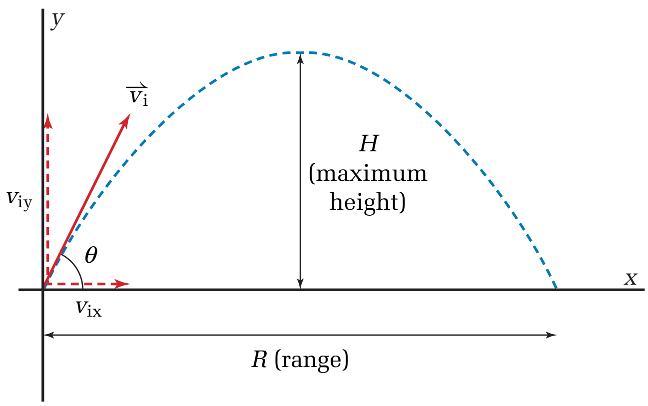


Figure 2.4 The maximum height, H, and the range, R, as well as the time of flight, T, are functions of the initial velocity, $\vec{v}_i$, the angle, θ, and the acceleration due to gravity, g.

Time of flight

- The time of flight ends when the projectile hits the ground. Since the height of the projectile is zero when it hits the ground, you can express this position as $\Delta y = 0$. Write the kinematic equation for vertical displacement and set $\Delta y = 0$.

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a\Delta t^2$$
$$0 = v_{iy}\Delta t + \frac{1}{2}a\Delta t^2$$

- Write the vertical component of the velocity in terms of the initial velocity and the angle θ. Then, substitute the expression into the equation above. Also, substitute $-g$ for the acceleration, a.

$$v_{iy} = |\vec{v}_i|\sin\theta$$
$$0 = (|\vec{v}_i|\sin\theta)\Delta t + \frac{1}{2}(-g)\Delta t^2$$

- Rearrange the equation to put the zero on the right-hand side and factor out a Δt.

$$\frac{1}{2}g\Delta t^2 - |\vec{v}_i|\Delta t\sin\theta = 0$$
$$\Delta t\left(\frac{1}{2}g\Delta t - |\vec{v}_i|\sin\theta\right) = 0$$

- If either factor is zero, the equation above is satisfied. Write the two solutions.

$$\Delta t = 0 \quad \text{or} \quad \frac{1}{2}g\Delta t = |\vec{v}_i|\sin\theta$$
$$\Delta t = \frac{2|\vec{v}_i|\sin\theta}{g}$$

- $\Delta t = 0$ represents the instant that the projectile was launched. Therefore, the second expression represents the time of flight, T, that the projectile spent in the air before it landed. Since T is a scalar, write the initial velocity without a vector symbol.

$$T = \frac{2v_i\sin\theta}{g}$$

Range

- The range is the horizontal distance that the projectile has travelled when it hits the ground. Write the equation for displacement in the horizontal direction.

$$\Delta x = v_{ix}\Delta t$$

- Write the expression for the horizontal component of velocity in terms of the initial velocity and the launch angle θ. Substitute this expression into the equation for the displacement above.

$$v_{ix} = |\vec{v}_i| \cos\theta$$
$$\Delta x = (|\vec{v}_i| \cos\theta)\Delta t$$

- Since the projectile is at the endpoint of its range, R, when $\Delta t = T$, substitute the expression for T into the equation and simplify. Since R is always in one dimension, omit the vector symbol for the initial velocity.

$$R = \frac{(v_i \cos\theta)(2v_i \sin\theta)}{g}$$
$$R = \frac{v_i^2 2\sin\theta\cos\theta}{g}$$

- Write the trigonometric identity for $2\sin\theta\cos\theta$ and substitute the simpler form into the equation.

$$2\sin\theta\cos\theta = \sin 2\theta$$
$$R = \frac{v_i^2 \sin 2\theta}{g}$$

Maximum height

- As a projectile rises, it slows its upward motion, stops, and then starts downward. Therefore, at its maximum height, its vertical component of velocity is zero. Write the kinematic equation that relates initial and final velocities, acceleration, and displacement and solve for displacement, Δy.

$$v_{fy}^2 = v_{iy}^2 + 2a\Delta y$$
$$0 = v_{iy}^2 + 2a\Delta y$$
$$2a\Delta y = -v_{iy}^2$$
$$\Delta y = \frac{-v_{iy}^2}{2a}$$

- Substitute the expression for initial vertical velocity in terms of the initial velocity and the launch angle, θ. Substitute $-g$ for a. Now Δy is the maximum height, H. Since H is always in one dimension, omit the vector symbol for the initial velocity.

$$H = \frac{-(|\vec{v}_i| \sin\theta)^2}{2(-g)}$$
$$H = \frac{v_i^2 \sin^2\theta}{2g}$$

These three relationships — time of flight, range, and the maximum height — allow you to make important predictions about projectile motion without performing calculations. For example, you can determine the launch angle that will give you the maximum range by inspecting the equation for range. Study the logic of the following steps.

- Inspect the equation for range.

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

- For a given initial velocity on the surface of Earth, the only variable is θ. Therefore, the term "$\sin 2\theta$" determines the maximum range. The largest value that the sine of any angle can achieve is 1.

$$\sin 2\theta = 1$$
$$R_{max} = \frac{v_i^2(1)}{g}$$

- For what angle, θ, is $\sin 2\theta = 1$? Recall that the angle for which the sine is 1 is 90°. Use this information to find θ.

$$\sin 2\theta = 1$$
$$2\theta = \sin^{-1} 1$$
$$2\theta = 90^\circ$$
$$\theta = 45^\circ$$

For any symmetrical trajectory, neglecting air friction, the launch angle that yields the greatest range is 45˚. Draw some conclusions of your own by answering the questions in the Conceptual Problems that follow.

Conceptual Problems

- Examine the equation for maximum height. For a given initial velocity, what launch angle would give a projectile the greatest height? What would be the shape of its trajectory?
- Examine the equation for time of flight. For a given initial velocity, what launch angle would give a projectile the greatest time of flight? Would this be a good angle for a football punter? Why?
- Consider the equation for range and a launch angle of 30˚. What other launch angle would yield a range exactly equal to that of the range for an angle of 30˚?
- Find another pair of launch angles (in addition to your answer to the above question) that would yield identical ranges.
- The acceleration due to gravity on the Moon is roughly one sixth of that on Earth ($g_{moon} = \frac{1}{6}g$). For a projectile with a given initial velocity, determine the time of flight, range, and maximum height on the Moon relative to those values on Earth.
- The general equation for a parabola is $y = Ax^2 + Bx + C$, where A, B, and C are constants. Start with the following equations of motion for a projectile and develop one equation in terms of Δx and Δy by eliminating Δt. Show that the resulting equation, in which Δy is a function of Δx, describes a parabola. Note that the values for the initial velocity (v_i) and launch angle (θ) are constants for a given trajectory.

$$\Delta x = v_i \Delta t \cos\theta$$
$$\Delta y = v_i \Delta t \sin\theta - \frac{1}{2}g\Delta t^2$$

SAMPLE PROBLEM

Analyzing a Kickoff

A player kicks a football for the opening kickoff. He gives the ball an initial velocity of 29 m/s at an angle of 69˚ with the horizontal. Neglecting friction, determine the ball's maximum height, hang time, and range.

Conceptualize the Problem

- A football field is level, so the trajectory of the ball is a *symmetrical parabola*.
- You can use the equations that were developed for symmetrical trajectories.
- "Hang time" is the time of flight of the ball.

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Identify the Goal

The maximum height, H, of the football
The time of flight, T, of the football
The range, R, of the football

Identify the Variables and Constants

Known	Implied	Unknown
$\lvert\vec{v}_i\rvert = 29\ \frac{\text{m}}{\text{s}}$	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	H
$\theta = 69°$		T
		R

Develop a Strategy

Use the equation for the maximum height of a symmetrical trajectory.

$$H = \frac{v_i^2 \sin^2\theta}{2g}$$

Substitute the numerical values and solve.

$$H = \frac{\left(29\ \frac{\text{m}}{\text{s}}\right)^2(\sin 69°)^2}{2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}$$

$$H = \frac{\left(841\ \frac{\text{m}^{\cancel{2}}}{\cancel{\text{s}^2}}\right)(0.871\ 57)}{19.62\ \frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}}$$

$$H = 37.359\ \text{m}$$

$$H \cong 37\ \text{m}$$

The maximum height the football reached was 37 m.

Use the equation for the time of flight of a symmetrical trajectory.

$$T = \frac{2v_i \sin\theta}{g}$$

Substitute the numerical values and solve.

$$T = \frac{2\left(29\ \frac{\text{m}}{\text{s}}\right)(\sin 69°)}{\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}$$

$$T = \frac{\left(58\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)(0.933\ 58)}{9.81\ \frac{\cancel{\text{m}}}{\text{s}^{\cancel{2}}}}$$

$$T = 5.5196\ \text{s}$$

$$T \cong 5.5\ \text{s}$$

The time of flight, or hang time, of the football was 5.5 s.

Use the equation for the range of a symmetrical trajectory.

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

Substitute the numerical values and solve.

$$R = \frac{\left(29\ \frac{\text{m}}{\text{s}}\right)^2(\sin 2(69°))}{9.81\ \frac{\text{m}}{\text{s}^2}}$$

$$R = \frac{\left(841\ \frac{\text{m}^{\cancel{2}}}{\cancel{\text{s}^2}}\right)(0.669\ 13)}{9.81\ \frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}}$$

$$R = 57.3637\ \text{m}$$

$$R \cong 57\ \text{m}$$

The football travelled 57 m.

Validate the Solution

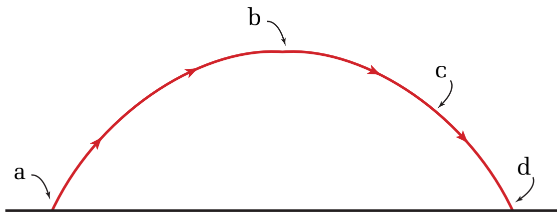
All of the values are reasonable for a football kickoff. In every case, the units cancel properly to give metres for the range and maximum height and seconds for the time of flight, or hang time.

PRACTICE PROBLEMS

13. A circus stunt person was launched as a human cannon ball over a Ferris wheel. His initial velocity was 24.8 m/s at an angle of 55°. (Neglect friction)
    (a) Where should the safety net be positioned?
    (b) If the Ferris wheel was placed halfway between the launch position and the safety net, what is the maximum height of the Ferris wheel over which the stunt person could travel?
    (c) How much time did the stunt person spend in the air?
14. You want to shoot a stone with a slingshot and hit a target on the ground 14.6 m away. If you give the stone an initial velocity of 12.5 m/s, neglecting friction, what must be the launch angle in order for the stone to hit the target? What would be the maximum height reached by the stone? What would be its time of flight?

2.1 Section Review

1. K/U Projectiles travel in two dimensions at the same time. Why is it possible to apply kinematic equations for one dimension to projectile motion?
2. K/U How does the analysis of projectiles launched at an angle differ from the analysis of projectiles launched horizontally?
3. C Explain why time is a particularly significant parameter when analyzing projectile motion.
4. C What can you infer about the velocity at each labelled point on the trajectory in this diagram?



5. C Imagine that you are solving a problem in projectile motion in which you are asked to find the time at which a projectile reaches a certain vertical position. When you solve the problem, you find two different positive values for time that both satisfy the conditions of the problem. Explain how this result is not only possible, but also logical.
6. K/U What properties of projectile motion must you apply when deriving an equation for the maximum height of a projectile?
7. K/U What properties of projectile motion must you apply when deriving an equation for the range of a projectile?
8. I Suppose you knew the maximum height reached by a projectile. Could you find its launch angle from this information alone? If not, what additional information would be required?